CLASSIFICATION OF DIGITIZED CONTOURS REPRESENTED BY SIGNATURE

Krzyzak, A. and El buaeshi, H.
Department of Computer Science
Concordia University
Montreal, Quebec H3G 1M8

ABSTRACT

A classification of discrete contours via signatures is studied. Two algorithms to compute the signature are developed. In the first algorithm a multidimensional sorting is used. The second algorithm is based on a simple geometrical considerations. Two types of signature are considered –the length signature and the area signature. Statistical features based on Fourier descriptors are derived from the signatures. In classification the k-NN algorithm is used with k and the size of the feature vector chosen experimentally. The algorithms are tested on the handwritten, totally unconstrained characters from Suen's data base and recognition success rates of 91% and 93% are achieved for the length and area signature respectively.

KEYWORDS: curve signature, Fourier descriptors, contour classification.

INTRODUCTION

In many applications of pattern recognition and digital image processing the shape of a simply connected object is represented by its outer contour. Many shape recognition techniques deal with the entire object boundary, silhouette, intensity profile or range map. They include such methods as Fourier descriptors of the object boundary [2, 6, 7, 9, 10, 11, 15, 16, 18], moments of the silhouette [3, 14], and circular autoregressive models [8]. Among different techniques Fourier descriptors and curve signatures are distinguished by the invariance to the standard shape transformations such as scaling, rotation and translation. Some functions of Fourier descriptors are also invariant to mirror reflections and changes in the starting point [9, 15]. In this paper we studv the shape recognition problem using Fourier descriptors (FD's) derived from the curve signatures. This approach combines the simplicity of curve signatures with invariance of Fourier descriptors. We use the length signature proposed by O'Rourke [13] as well introducing an area signature. The latter is shown to be more robust with respect to the shape distortion. Efficient algorithms for computation of curve signatures are proposed and implemented. In shape classification features based on Fourier descriptors are used. Fourier descriptors are derived from the signature and are characterized by invariance to affine transformations, mirror reflections and changes in the starting point. We study several versions of Fourier descriptors and compare them. The shape representation methods proposed in this paper are tested in unconstrained handwritten numeral recognition. Basic components of the implemented recognition system are shown in Figure 1.

The feature extraction module consists of two sub-modules: signature module and Fourier descriptors module. The algorithm based on the geometrical considerations is used to compute the length/area signature of the numeral. The Fourier descriptors module computes the FD's of the numerals from their signatures. The classification module is divided into two sub-modules as follows:

1) k-NN sub-module to find the k-nearest neighbor using branch and bound algorithm [4].
2) signature verification sub-module to separate distinct numerals with similar FD's.

In sub-module 1 FD's are used alone while in sub-module 2 classification is based on the signature.

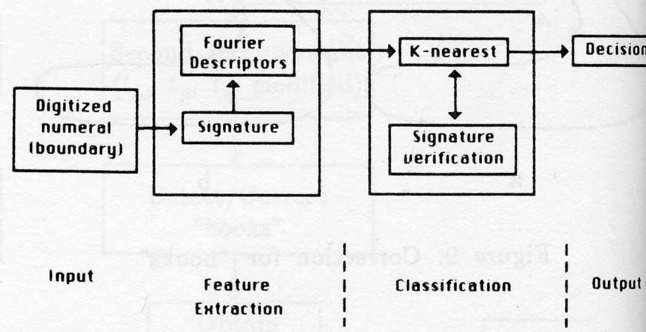

Figure 1. The basic components of the implemented character recognition system.

CURVE SIGNATURE

Among feature extraction techniques used in shape recognition the most popular ones are: Fourier descriptors [7, 15, 18], boundary line encoding [10], polygonal approximation and directional and curvature feature extraction (see [17] for review). In this and the following sections feature extraction methods based on the signature and Fourier descriptors will be presented.

A signature of a plane curve is defined by O'Rourke [13]. Let Γ be a continuous, directed curve in the Euclidean plane and parametrized by its arc length t. The signature $S(t)$ of Γ is the function which associates each point t of Γ with the length of Γ which is on or to the left of a tangent line at point t –Figure 2. We also consider the alternative version of the signature defined by taking the area to the left of a tangent line instead of the length to the left –Figure 3.

Among advantages of the curve signature we list its simplicity (the signature of a polygon is represented by the histogram) and invariance to shifts and rotations (as long as the starting point is maintained.) The main demerit of the signature is that it does not uniquely identify the curve it is derived from, except for rectilinear curves [13]. For instance all convex curves of unit length are mapped on the constant signature at one or zero depending on the orientation. Nevertheless, curve signature is a valuable tool in shape recognition as it is shown in the following sections.

In this paper we consider the normalized signatures obtained by dividing the value of the signature at point t by the total length of the curve or area bounded by the curve and by taking $S^*(t) = S(Lt)$, where L is the length of Γ and $t \in [0, 1]$. We use the following formula to calculate the area for the polygonal curves with vertices $V_1 = (x_1, y_1), \cdots, V_n = (x_n, y_n)$

$$\text{area} = \frac{1}{2} \mid \sum_{i=1}^{n} x_i (y_{i+1} - y_{i-1}) \mid \tag{1}$$

where subscripts are reduced modulo n and $y_0 = 0$.

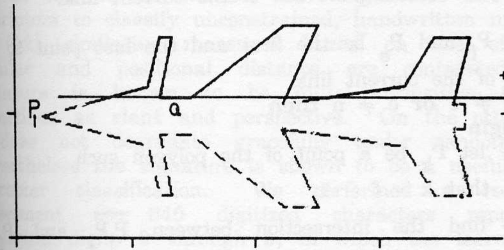

Figure 2. The length of continuous line is the signature value at link Q of the polygon.

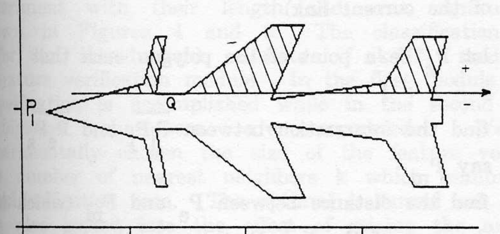

Figure 3. The filled area is the signature value at link Q of the polygon.

FOURIER DESCRIPTORS

Fourier descriptors of plane curve were first introduced by Cosgriff [1] and subsequently used by a number of researchers [2, 7, 9, 10, 11, 15, 16, 18]. It is well known that the similar shapes regardless of their size and location usually fall in the same cluster in the Fourier descriptors space using the Euclidean metric distance. We propose new Fourier descriptors which are derived from the coefficients of a Fourier series corresponding to either the length or the area signature. We only consider polygonal curves.

Let us assume that the curve Γ has n vertices V_0, $\cdots, V_n = V_0$ and that the edge (V_{j-1}, V_j) has the length $\Delta\ell$. Signature is a step function given by equation $S^*(t) = \sum_{j=1}^{n} c_j I_{A_j}(Lt)$, where $A_j = [\ell_{j-1}, \ell_j]$, $\ell_j = \sum_{k=1}^{j} \Delta\ell_k$ and $I_A(\cdot)$ is the indicator function of a set A. Coefficients c_j denote either the length of Γ to the left of the edge (V_{j-1}, V_j) in case of the length signature or the area of Γ to the left of the edge (V_{j-1}, V_j) in case of the area signature. Fourier coefficients of $S^*(t)$ are given by the following equation:

$$a_m = \frac{1}{\pi m} \sum_{j=1}^{n} c_j \exp[-i(m2\pi\ell_{j-1}/L - \frac{\pi}{2})] \cdot$$

$$\exp[-i(m2\pi\Delta\ell_j/L) - 1],$$

$$a_0 = \frac{1}{L} \sum_{j=1}^{n} c_j \Delta\ell_j$$

where i is equal to $\sqrt{-1}$.
The Fourier coefficients a_m may be expressed in the magnitude-phase form as

$$a_m = A_m \exp(i\alpha_m)$$

where

$$A_m = (\mathrm{Re}^2 a_m + \mathrm{Im}^2 a_m)^{1/2}, \tag{2}$$

$$\mathrm{Re}\, a_m = \frac{1}{m\pi} \sum_{j=1}^{n} c_j \sin[\frac{m\pi}{L}(2\ell_{j-1} + \Delta\ell_j)] \cdot$$

$$\cos(\frac{m\pi}{L}\Delta\ell_j),$$

$$\mathrm{Im}\, a_m = \frac{-1}{m\pi} \sum_{j=1}^{n} c_j \sin[\frac{m\pi}{L}(2\ell_{j-1} + \Delta\ell_j)] \cdot$$

$$\sin(\frac{m\pi}{L}\Delta\ell_j),$$

$$\alpha_m = \arctan(\mathrm{Im}\, a_m / \mathrm{Re}\, a_m).$$

The Fourier descriptors are defined as $\{A_m, \alpha_m\}_1^n$. It can be easily shown that A_m's are not only invariant to translations, rotations and scaling (that follows from the definition of $S^*(t)$ and eq. (2)) but also to changes in the starting point and mirror reflections. On the other hand the last transformation affects the phase angles α_m's which can be used to distinguish between curves which are mirror image versions of each other, for example characters 5 and 2. We will address the problem of the choice of the number of FD's in the following sections.

THE ALGORITHMS FOR SIGNATURE COMPUTATION

In this section we present the efficient algorithms to calculate the signature of a plane curve. Algorithms 1 and 2 find the vertices to the left of the current link of the polygon. The first algorithm uses the multidimensional sorting [5]. The second algorithm uses the simple geometrical properties of a polygon. Both algorithms use algorithm 3 to calculate the length or the area to the left of the current link of a polygon. These algorithms were written in Pascal language and executed on the MicroVax II workstation. Algorithm 2 is on average four times faster than algoritm 1.

Algorithm 1.
Description. The algorithm presented here uses the multidimensional sorting of [5] as follows: translate the coordinate system origin to a polygonal vertex P_i, i=1, $\cdots$,n, reflect each vertex $P_j \neq P_i$, with respect to P_i obtaining a vertex P_{n+j}, and sort the vertices and their reflections circularly about P_i, the order within each ray about P_i being immaterial. In order to obtain the vertices to the left of the current origin P_i we only have to identify the original vertices from the ray $\overline{P_iP_j}$ to the ray

$\overline{P_iP_{j+n}}$ counterclockwise.

Pseudo-code.

Input: $P_i = (x_i, y_i)$, $1 \le i \le n$.

Output: Signature(i).length/area, $1 \le i \le n$.

1) **For** i=1, ···, n **do** step 2 to step 8.
2) **For** j=1, ···,n **do** $u_j = x_j - x_i$, $v_j = y_j - y_i$.
3) For j=1, ···,n, if $(u_j, v_j) \neq (0,0)$ call j "good".
4) **For** every good j=1, ···,n let $u_{j+n} = -u_j$, $v_{j+n} = -v_j$, and $s_{j+n} = v_j/u_j$, provided that $u_j \neq 0.0$ (reflect P_j with respect to P_i and find the slope of $\overline{P_iP_j}$).
5) Sort the indices $\{j \mid j$ is good $\} \cup \{n+j \mid j$ is good$\}$ into the subsets as follows
   - $\{j \mid u_j > 0;\ s_j$ -key$\}$
   - $\{j \mid u_j = 0,\ v_j > 0\}$
   - $\{j \mid u_j < 0;\ s_j$ -key$\}$
   - $\{j \mid u_j = 0,\ v_j < 0\}$.
6) Find the vertices to the left of the current link $\overline{P_iP_j}$ by taking the original vertices going counterclockwise from the ray containing P_j to the ray containing the reflection P_{j+n}.
7) Sort the vertices to the left of the current link using the index as the key.
8) Signature[i].length = the length of the polygon to the left of the current link (use algorithm 3).
   Signature[i].area = the area bounded by the points on and to the left of the current link (use eq. (1) and algorithm 3).

Algorithm 2.

Description. This algorithm is based on the simple geometrical properties of a polygon. It proceeds as follows for each link of the polygon the coordinate system is rotated and translated so that the origin coincides with the starting vertex of the link. The x-axis is then aligned with the polygon link. The vertices (x, y) to the left of the link are the ones with $y \ge 0.0$.

Pseudo-code.

Input: $P_i = (x_i, y_i)$, $1 \le i \le n$.

Output: Signature(i).length/area $1 \le i \le n$.

1) **For** i=1, ···,n **do** step 2 to step 6.
2) Let j = i + 1.
3) Translate the coordinate system OXY by the vector $\overline{OP_i}$ to the coordinate system $P_iX'Y'$ as follows:
   for m = 1, ···, n **do**
   $x'_m = x_m - x_i$, $y'_m = y_m - y_i$.
4) Rotate the coordinate system $P_iX'Y'$ through an angle θ about the origin P_i to the new coordinate system $P_iX''Y''$ as follows:
   for m = 1, ···, n **do**
   $x''_m = x'_m \cos\theta + y'_i \sin\theta$
   $y''_m = -x'_m \sin\theta + y'_i \cos\theta$.
5) Find and sort all vertices $y''_m \ge 0.0$ to the left of the current link $\overline{P_iP_j}$ of the polygon using the index as a key.
6) Signature[i].length = the length of the curve to the left of current link (compute the length using algorithm 3).

Signature[i].area = the area bounded by the points on and to the left of the current link (use eq. (1) and algorithm 3).

Algorithm 3.

This algorithm finds:
1) the intersection points between the ray containing the current link and the other links of the polygon.
2) the length of the line segments connecting the points on and to the left of the current link.

Pseudo-code.

Input: $\overline{P_iP_j}$, vertices to the left of the current link, number of vertices to the left of the current link. where $P_0 = P_n$, and $P_1 = P_{n+1}$.

Output: the total length of the line segments or the area to the left of the current link.

1) Let P_f and P_e be the first and the last point to the left of the current link.
2) **if** $f \neq 1$ **or** $e \neq n$ **then**
   begin
   - let P_k be a point of the polygon such that k = f − 1
   - find the intersection between $\overline{P_iP_j}$ and $\overline{P_fP_k}$ say P_w
   - find the distance between P_f and P_w (which is the part of the link that lies to the left of the current link)
   - let P_z be a point of the polygon such that z = e + 1
   - find the intersection between $\overline{P_iP_j}$ and $\overline{P_eP_z}$ say P_m
   - find the distance between P_e and P_m (which is the part of the link that lies to the left of the current link)

   end
3) **else** find the distance between P_1 and P_n.
4) Compute the length of the line segments connecting the vertices P_f, ···, P_e as follows:
   - let P_t and P_r be two consecutive vertices to the left of the current link such that t < r
   - **if** t − r = 1 **then** find the distance between P_t and P_r
   - **if** t − r ≠ 1 **then**
   begin
     - find the intersection between $\overline{P_iP_j}$ and $\overline{P_tP_{t+1}}$ say P_o
     - find the distance between P_t and P_o (which is the part of the link lies to the left of the current link)
     - find the intersection between $\overline{P_iP_j}$ and $\overline{P_rP_{r-1}}$ say P_s
     - find the distance between P_r and P_s (which is the part of the link lies to the left of the current link)

   end.
5) Total-length = the sum of the length of the line segments computed in step 2 or step 3 and step 4.

Comments

Step 2 computes the length of the first and the last line segment to the left of the current link, when not both vertices P_1 and P_n are to the left of the current link.

In order to calculate area to the left of the current link find in steps 1 to 4 the intersection points between the ray containing the current link and the other links of the polygon. Put these points in their toplogical order with respect to the order of other vertices. Change step 5 to find the area enclosed by the polygon defined by the vertices found in steps 1 through 4 using eq. (1).

CLASSIFICATION OF HANDWRITTEN CHARACTERS

In this section we use curve signatures and Fourier descriptors to classify unconstrained, handwritten numerals. In [12] similarity measures based on the signature, angular and positional distance are contrasted. The signature is known to be quite insensitive to such distortions as slant and perspective. On the other hand it does not degradate gracefully under random noise. Nevertheless the signature is shown to be a useful tool in character classification. We performed a recognition experiment on 840 digitized characters representing numerical digits 0 through 9, in which the learning and testing sequences consisted of 461 and 379 characters respectively. These characters represented a small subset of Suen's data base [10] of handwritten characters of 30 different styles. The samples of characters used in the experiment with their length and area signatures are shown in Figures 4 and 5. The classification module (refer to introduction) consists of k-NN module and signature verification module. In the first module a rough classification is accomplished while in the second one the ambiguous characters are seperated. We have experimentally chosen the size of the feature vector and the number of nearest neighbors k which minimized the misclassification rate. The optimal k turned out to be 5. We also looked into the effect of mixing the amplitudes A_m and the phase angles α_m in the feature vector. We discovered that the phase angles alone behaved poorly (24% succes rate for the length signature and 39% for the area signature), so we decided to use only FD's amplitudes as the features in the k-NN module. The optimal number of A_m's amplitudes was 6 for the length signature and 9 for the area signature. With that choice of parameters we achieved 85.5% overall recognition rates for both the length and the area signature (see confusion matrices in Table 1 and Table 2). These results compare favourably with the literature. Using different FD's Persoon and Fu [15] obtained 84.6% rate and Shridhar and Badreldin [16] 66% rate on the set of carefully selected handwritten characters. A glance at the confusion matrices reveals that in many cases 6's were misclassified as 9's and 5's as 2's. The reason for this phenomenon lies in the rotational and mirror reflection invariance of A_m's in eq. (2). To remedy the problem we introduced a differentiation scheme.

Differentiation scheme.

In order to separate the characters which are mirror reflections or rotations of one another (and consequently have identical A_m's) we use a differentiation scheme based on the signature itself. The signature is clearly sensitive to reflections and changes in the starting point therefore we use it to find the set of discriminating inequalities to distinguish between easily confused characters. The inequalities in Table 3 and Table 4 are found experimentally after tesing the large set of characters from

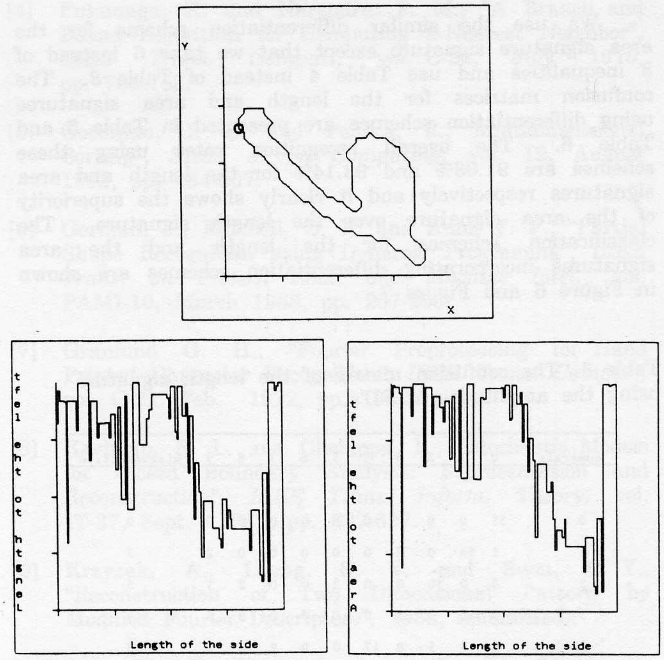

Figure 4. The numeral 6 and its length and area signatures.

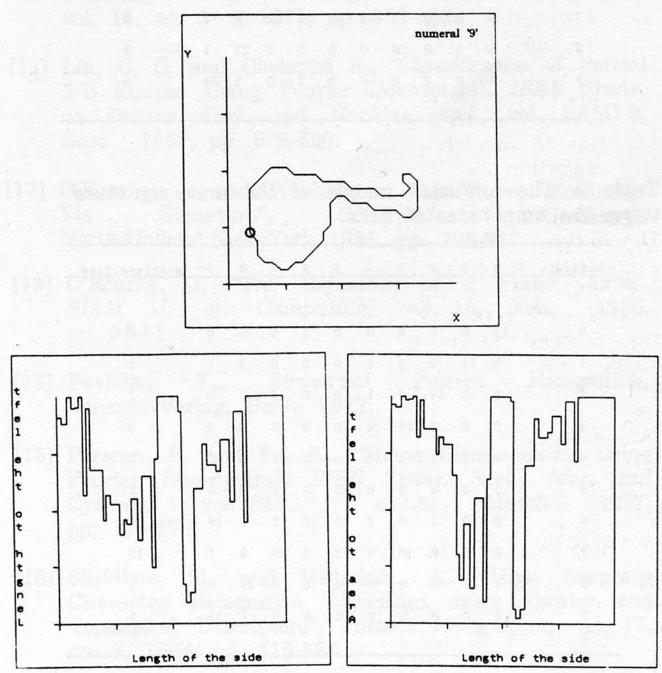

Figure 5. The numeral 9 and its length and area signatures.

the learning sequence. For the length signature we use the follwing differentiation scheme.

- For each character divide the domain of the signature (step function) into four equal intervals and calculate the coresponding areas under the signature, $q_1, \cdots, q_4$.
- For each category count the number of times $q_i < q_j$, where $1 \leq i,j \leq 4,\ i < j$.
- Choose the best 3 inequalities which have the highest count number (Table 3) to be the template for the category.
- Assign the character to a class with the maximum number of satisfactions of the inequalities in Table 3.

We use the similar differentiation scheme for the area signature signature except that we take 6 instead of 3 inequalities and use Table 4 instead of Table 3. The confusion matrices for the length and area signatures using differentiation schemes are presented in Table 5 and Table 6. The overall recognition rates using these schemes are 91.03% and 93.14% for the length and area signatures respectively and it clearly shows the superiority of the area signature over the length signature. The classification schemes for the length and the area signatures incorporating differentiation schemes are shown in Figure 6 and Figure 7.

Table 1. The confusion matrix of the length signature using the amplitudes of FD's.

numerals	0	1	2	3	4	5	6	7	8	9	misclassified
0	55	0	0	0	0	0	0	0	0	0	0
1	1	53	0	0	0	0	0	0	0	2	3
2	0	0	30	0	0	2	0	0	2	1	5
3	0	0	0	55	0	0	0	0	0	0	0
4	0	4	5	0	17	0	0	2	0	1	12
5	0	0	3	0	0	23	0	0	0	0	3
6	0	0	0	0	0	0	22	0	0	15	15
7	0	0	0	0	2	0	6	12	0	0	8
8	1	3	0	0	0	0	0	0	23	1	5
9	1	0	0	0	0	0	2	0	1	34	4

Table 2. The confusion matrix of the area signature using the amplitudes of FD's.

numerals	0	1	2	3	4	5	6	7	8	9	misclassified
0	55	0	0	0	0	0	0	0	0	0	0
1	0	51	0	0	1	0	0	0	2	1	5
2	0	0	34	0	1	0	0	0	1	0	1
3	0	0	0	55	0	0	0	0	0	0	0
4	0	3	2	1	22	0	0	1	0	0	7
5	0	0	4	0	0	22	0	0	0	0	4
6	0	0	2	0	0	0	17	0	0	18	20
7	0	0	2	0	5	0	2	10	0	1	10
8	2	1	0	0	0	0	0	0	25	0	3
9	0	0	0	0	0	0	4	0	1	33	5

Table 3. The differentiating inequalities for the length signature

character	inequalities		
2	$q_2<q_3$,	$q_2<q_4$,	$q_3<q_4$
4	$q_1<q_2$,	$q_1<q_3$,	$q_1<q_4$
5	$q_1<q_2$,	$q_2<q_3$,	$q_2<q_4$
6	$q_1>q_4$,	$q_2>q_3$,	$q_2>q_4$
7	$q_1<q_4$,	$q_2<q_3$,	$q_2<q_4$
9	$q_2<q_3$,	$q_2<q_4$,	$q_3<q_4$

Table 4. The differentiating inequalities for the area signature

character	inequalities
2	$q_1<q_2$, $q_1<q_3$, $q_1<q_4$, $q_2<q_3$, $q_2<q_4$, $q_3<q_4$
2	$q_1<q_2$, $q_1<q_3$, $q_1<q_4$, $q_2<q_3$, $q_2<q_4$, $q_3<q_4$
4	$q_1<q_2$, $q_1<q_3$, $q_1<q_4$, $q_2>q_3$, $q_2>q_4$, $q_3<q_4$
5	$q_1>q_2$, $q_1<q_3$, $q_1>q_4$, $q_2<q_3$, $q_2<q_4$, $q_3>q_4$
6	$q_1>q_2$, $q_1>q_3$, $q_1>q_4$, $q_2>q_3$, $q_2>q_4$, $q_3>q_4$
7	$q_1>q_2$, $q_1<q_3$, $q_1<q_4$, $q_2<q_3$, $q_2<q_4$, $q_3<q_4$
9	$q_1>q_2$, $q_1>q_3$, $q_1>q_4$, $q_2<q_3$, $q_2<q_4$, $q_3<q_4$

Table 5. The confusion matrix for the length signature using the differentiation scheme (from Table 3).

numerals	0	1	2	3	4	5	6	7	8	9	misclassified
0	55	0	0	0	0	0	0	0	0	0	0
1	1	53	0	0	1	0	0	0	0	1	3
2	0	0	32	0	1	0	0	0	2	0	3
3	0	0	0	55	0	0	0	0	0	0	0
4	0	4	4	0	18	1	0	2	0	0	11
5	0	0	3	0	0	23	0	0	0	0	3
6	0	0	0	0	0	0	37	0	0	0	0
7	0	0	0	0	2	0	0	14	0	4	6
8	1	3	0	0	0	0	1	0	23	0	5
9	1	0	0	0	0	0	1	0	1	35	3

Table 6. The confusion matrix for the area signature using the differentiation scheme (from Table 4).

numerals	0	1	2	3	4	5	6	7	8	9	misclassified
0	55	0	0	0	0	0	0	0	0	0	0
1	0	51	0	0	1	0	0	0	2	1	5
2	0	0	34	0	1	0	0	0	1	0	1
3	0	0	0	55	0	0	0	0	0	0	0
4	0	3	2	1	22	0	0	1	0	0	7
5	0	0	1	0	0	25	0	0	0	0	1
6	0	0	0	0	0	2	35	0	0	0	2
7	0	0	2	0	0	0	0	15	0	3	5
8	2	1	0	0	0	0	0	0	25	0	3
9	0	0	0	0	0	0	1	0	1	36	2

CONCLUSION

In the paper two kinds of curve signatures are discussed and implemented. Fourier descriptors which are invariant to affine transformations (translations, rotations and scaling), mirror reflections and changes in starting point are derived for both signatures. To distinguish between contours which are rotations or mirror reflections of one another the differentiation scheme is implemented. We note that the area signature performs better than the length signature both with respect to classification rate and the computational complexity. It is interesting to see how the random noise and other nonlinear distortions affect the signature performance. Another open problem is a fast parallel implementation of the signature computation algorithm.

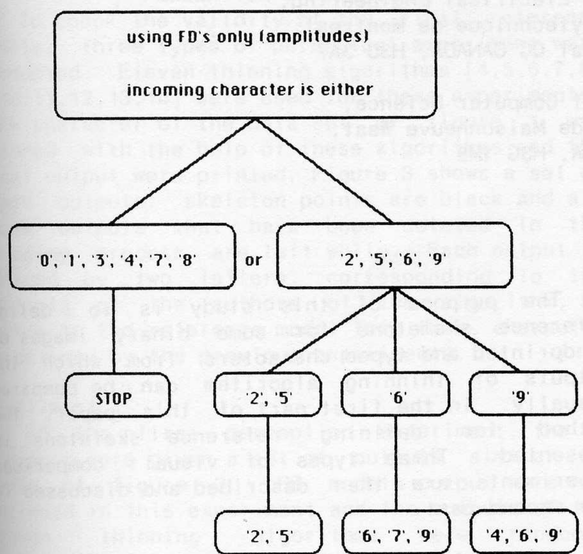

Figure 6. Differentiation scheme for the length signature.

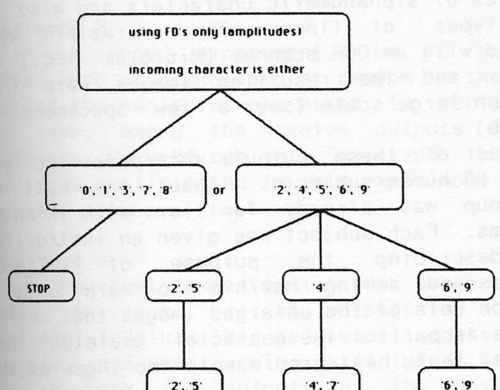

Figure 7. Differentiation scheme for the area signature.

REFERENCES

[1] Cosgriff, R., "Indentification of Shape", *Ohio State Univ. Res. Foundation*, Columbus, Rep. 820-11, ASTIA AD 254 742, Dec. 1960.

[2] Dekking, F. M. and Otterloo, P. J. V., "Fourier Coding and Reconstruction of Complicated Contours", *IEEE Trans. on Syst., Man and Cybern.*, vol. SMC-16, no. 3, May/June 1986, pp. 395-404.

[3] Dudani, S. A., et al. "Aircrarft Identification by Moment Invariants", *IEEE Trans. Comput.*, vol. 26, Jan. 1977, pp. 39-46.

[4] Fukunaga, K. and Narendra, P. M., "A Branch and Bound Algorithm for Computing k-Nearest Neighbor", *IEEE Trans. Comput.*, vol. C-24, July 1975, pp. 750-753.

[5] Goodman, J. E. and Pollack, R., "Multidimensional Sorting", *SIAM J. on Computing*, vol. 12, August 1983, pp. 484-507.

[6] Gorman, J., Mitchell, O. R. and Kuhl, F. P., "Partial Shape Recognition using Dynamic Programing", *IEEE Trans. on Pattern Anal. and Machine Intel.*, vol. PAMI-10, March 1988, pp. 257-266.

[7] Granlund G. H., "Fourier Preprocessing for Hand Printed Character Recognition", *IEEE Trans. Comput.*, vol. C-21, Feb. 1972, pp. 195-201.

[8] Kashyap, R. L. and Chelappa, R., "Stochastic Models for Closed Boundary Analysis: Representation and Reconstruction", *IEEE Trans. Inform. Theory*, vol. IT-27, Sept. 1981, pp. 627-637.

[9] Krzyzak, A., Leung, S. Y. and Suen, C. Y., "Reconstruction of Two Dimensional Pattern by Modified Fourier Descriptors", 1988, (submitted).

[10] Lai, M. T. Y. and Suen, C. Y., "Automatic Recognition of Charaters by Fourier Descriptors and Boundary Line Encodings", *Pattern Recognition*, vol. 14, no. 1–6, 1981, pp. 383-393.

[11] Lin, C. C. and Chelappa R., "Classification of Partial 2-D Shapes Using Fourier Descriptors", *IEEE Trans. on Pattern Anal. and Machine Intel.*, vol. PAMI-9, Sept. 1987, pp. 686-690.

[12] O'Rourke, J. and Washington, R., "Curve Similarity Via Signature", *Computaional Geometry*, North-Holland:New York 1985, pp. 295-317.

[13] O'Rourke, J. "The Signature of a Plane Curve", *SIAM J. on Computing*, vol. 15, Feb. 1986, pp. 34-51.

[14] Pavlidis, T., *Structrual Pattern Recognition*, Springer-Verlag, Berlin 1977.

[15] Persoon, E. and Fu, K., "Shape Discrimination using Fourier Descriptors", *IEEE Trans. Syst., Man and Cybern.*, vol. SMC-7, no. 3, March 1977, pp. 170-179.

[16] Shridhar, M. and Badreldin, A., "High Accuracy Character Recognition Algorithm using Fourier and Topological Descriptors", *Pattern Recognition*, vol. 17, no. 5, 1984, pp. 515-524.

[17] Suen, C. Y. and Berthod, M. and Mori, S., "Advances in Recognition of Handprinted Characters", *Proc. IEEE, vol. 68, 1980,* pp. 463-478.

[18] Zahn, C. T. and Roskies, R. Z., "Fourier Descriptors for Plane Closed Curves", *IEEE Trans. Comput.*, vol. C-21, March 1972, pp. 269-281.